Evolutionary Game Theory and Electoral Dynamics.

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7th May 2018.

Abstract: It is argued that there is a fundamentally different dynamic between typical electoral competition as modelled by the Median Voter Theorem and the EU Referendum and again by a typical large majority/landslide situation. It is proposed that the replicator dynamic from continuous evolutionary game theory can illuminate this. This informal exposition of the ideas avoids mathematics.

Key concepts: EU Referendum, Median Voter Theorem , Evolutionary Game Theory Replicator Dynamics

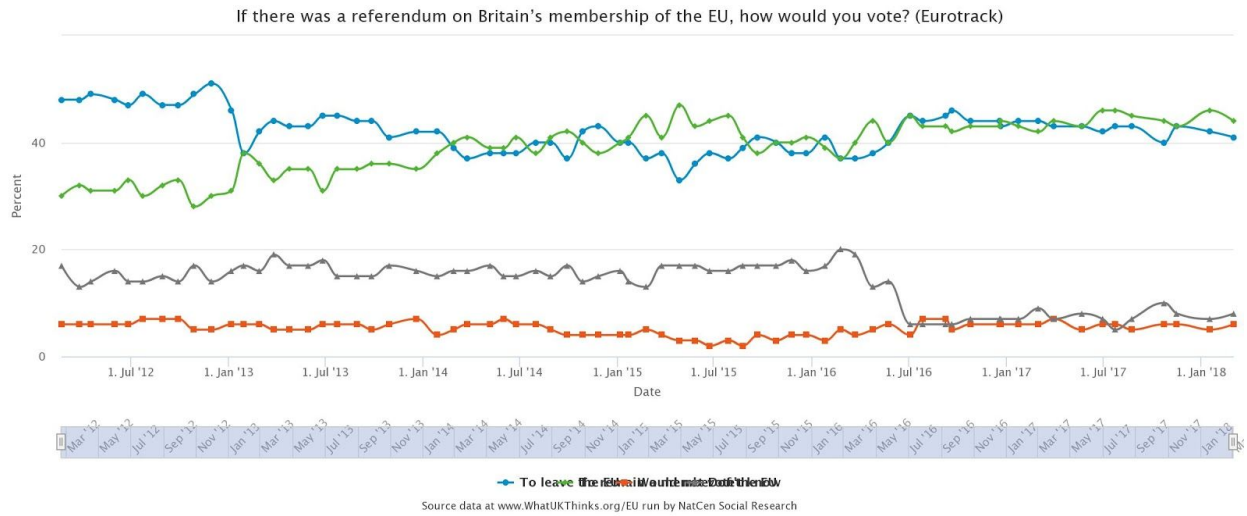
One of the claims of the 'genius' of Democracy is its ability to constrain conflict. Within reason, democratic systems impose a limitation on social conflict which,on the whole, delivers compromise rather than conflict. A simple analytical model which reflects this is the Median Voter Theorem. This is essentially a game played on a single issue dimension by two candidates seeking electoral advantage. The assumption is that voters are evenly arranged along the dimension in terms of their positioning (very positive, fairly positive, fairly weakly positive, indifferent, fairly weakly negative, fairly negative ..and so on). It is always in the candidate's interest to position him/herself as close as possible to the centre. Moderate positions, in terms of capturing voter numbers, dominate extreme positions and they in turn are dominated by yet more moderate ones. Domination here is used in the game theoretic sense. A payoff is (strictly) better if its payoffs are strictly better.

On this view elections are always "won from the centre" and political parties "stealing each others clothes (i.e., policies)" is fairly common. Parties seek advantage by encroaching on each other's base and minimising the differences between themselves.

An example from economics, oligopoly theory is the competition between Coke and Pepsi where both sustain a continuing equilibrium hold a large share of the market. To most people the the two products are close to being un-differentiable. The Coke Pepsi duopoly and the MVT are examples of the minimum difference principle.

The recent referendum on continuing membership, or not, of the European Union, in the UK delivered a different sort of result. There was no compromise at all. Voters, the nation, were

completely split, more or less fifty/fifty(52%, 48%). Subsequent polling evidence, the average of monthly polls plotted in the chart below show the split has changed little in the interim since then.



The top two lines in the chart, illustrates the sustained nature of the split. Somewhat analogous to trench warfare in WW1 Small advances and retreats but the basis positions don't change much. No sign of a compromise. Moreover this split has cross cut the major parties and is arguably along social conservative / social liberal rather than economic class lines. It has had an enormously debilitating influence on the UK political system causing widespread paralysis on both major political parties (lack of a convincing majority coupled with fear of electoral consequences) The Government and the Cabinet. Political leadership has responded with vacillation and fudge on the issue.

Compared with the Median Voter 'game' the game here was for both sides, particularly during the referendum, to seek to reinforce their own side rather than ingratiate with the opposition. The situation somewhat reminiscent of the (long running) war between Lilliput and Blefuscu On the right way (up) to eat an egg. It has roughly the structure of a "coordination" game. Payoffs go to those who do the same ("all drive on the same side") not those who compromise.

Evolutionary game theory in its dynamic[1] (Taylor Jonckers) form provides a selection of dynamics which arguably throws light on these two situations as well as the convention of large electoral gains, "sweeping to power" and so on. I am going to deal here with the simple situation of two players only.

[1] As I stress later, EVGT is quite unlike GT in that it doesn't deal with rational actors. Early developments Maynard Smith, Price dealt with a static picture and the principal result Evolutionary Stability was essentially a generalisation of the main stability solution on GT, Nash Equilibrium. Taylor and J... introduce the replicator equation based upon differential equations and dynamic systems theory.

Since this is a paper aimed at a non specialist audience and since there is nothing novel in the mathematics I am going to motivate the so called replicator dynamics informally, without mathematics and with a very informal, and short!, digresion.

Take first a single species. This is routinely modelled by the logistic curve.[2] This is based upon the idea that growth is not completely unconstrained but will be ultimately checked by the environment. The environment has a "carrying capacity" for the species which cannot be exceeded. There are only just so many rabbits that can feed off of a field of grass.

The logistic model posits a rate of growth which depends upon the size of the population, as the size of the population increases so the growth rate speeds up. It also incorporates a "brake". As the growing population gets closer and closer to the carrying capacity (the total limit that the environment is able to support) the rate the growth rate will slow, stop and start to decline.

There are two static points, or equilibria, where the rate of growth will be zero. One at the beginning, where the population is growing rapidly (the field is empty and the rabbits are breeding,well, like rabbits) and the the other equilibrium, when the population size has reached the carrying capacity. The first is unstable, when the rabbit population grows, it grows. When however it reaches its carrying capacity limit it stops growing, it stays there. This is a stable equilibrium.

One can take this basic model and apply it not to rabbits, or sheep eating grass but to political parties converting voters, I am ignoring here the complication of voters leaving allegiance to a political party but I am assuming that the appeal of a particular idea or policy is limited to its carrying capacity, it will only be endorsed by a percentage of the electorate. Another way to think about it is in terms of 'memes'(Dawkins and Dennett) in the original sense of ideas or concepts which can ensure their survival in a competitive electoral environment by "infecting " individual voters (in the recent past it certainly seems that way!) with fitness equating to the size of the infected group. The group of voters that the meme has "captured".

That is the one meme growth story. It generalises fairly readily to competition between two memes.

If there are two species competing on exactly the same environment. Rabbits and Sheep both competing for the same field of grass, or competitive memes for the same group of voters then here there are two logistic processes running in parallel, except that both species are eating each others lunch. Literally.

[2] See, for example Strogatz: Non-Linear Dynamics and Chaos 1994. A more informal treatment can be found in Flake: The Computational Beauty of Nature MIT (Bradford) 1998.

As before the growth of each species depends upon its size, but the braking term which previously reflected the carrying capacity now reflects both the carrying capacity restriction plus the size of the other specie population plus a factor measuring the interaction between the two.

For the logistic process there are two equilibrium points one stable and the other not. When two logistic processes are modified and put together to form the Lotka Volterra competitive model[3], there is a competition between two growth rates, if one species succeeds in outcompeting the other on growth then obviously its own rate increases, but also the "brake" on the other species grow also increases. So, a double benefit.

If the breaking effect of the carrying capacity and the effects on the competitor are weak then there will be a stable equilibrium point at the centre. This is a dynamic version of the median voter situation.

At this stable equilibrium the two competing memes will be be less than their total carrying capacity. That is they will have expanded less than they would have unconstrained by competition

So with this as background we can turn to the replicator dynamic. The Replicator equation can be derived from the Lotka Volterra competitive system[4]
and the rationale is very similar. This treats of the competitive strategies between, i.e., rate of growth or improvement of fitness, as the difference between the fitness of the given phenotype (meme) as compared with the average fitness of all.

The equilibrium is the average fitness where there is no difference between any given phenotype fitness and the average and therefore the rate of growth of each is zero (so equilibrium).

One important difference between the replicator equation and the Lotka Volterra equations is that the latter is in the context of evolutionary game theory[5]. The parameters which governed growth or limitations to growth in the L_V situation In the evolutionary context, become payoffs in the game, or competitive interaction between the two memes. This is a 2X2 table showing

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[3] Accounts of the Lotka-Volterra competitive model can be found in: Strogatz op cit.Kemeny and Snell Mathematical Models in the Social Sciences Blaisdell 1962 (this also covers the better known predator prey model) Maynard Smith: Mathematical Ideas in Biology Cambridge 1968. This also covers the logistic model.

[4] Roughly speaking see e.g., Bomze: Lotka-Volterra Equation and Replicator Dynamics. Biological Cybernetics 1983

[5] Political scientists are likely to have some familiarity with Game Theory, less likely with Evolutionary Game Theory. It is important to stress that the latter does not presuppose voter rationality. Rather proportions of voters reflect strategies, or genotypes or memes or "common opinions" and it is these that do the competing not rational players making pure or mixed selection of strategies.

the mutual impact of memes which,in turn, determines their fitness.The game is played repeatedly amongst random members of the population each of which is infected by a different meme or views. Thus the payoffs determine the results of each interaction and these drive the dynamic. The interaction between the two memes is defined as a payoff matrix.

I have mentioned that the median voter game can be expressed as a coalitional game, actually somewhat analogous to “Battle of the Sexs” followin the Mimimum difference principle. That the Referendum can also be expressed as a game game in analogous fashion but here the game is a “Coordination game” Both of these games plus the more straightforward case where one side completely dominates the other are outcomes of the replicator dynamic played between two competitors.

If the situation covers only two competing players, or memes, or parties expressing memes then the replicator dynamic reduces to a single equation whose graph (phase portrait) is governed by the relative strengths of the payoffs.

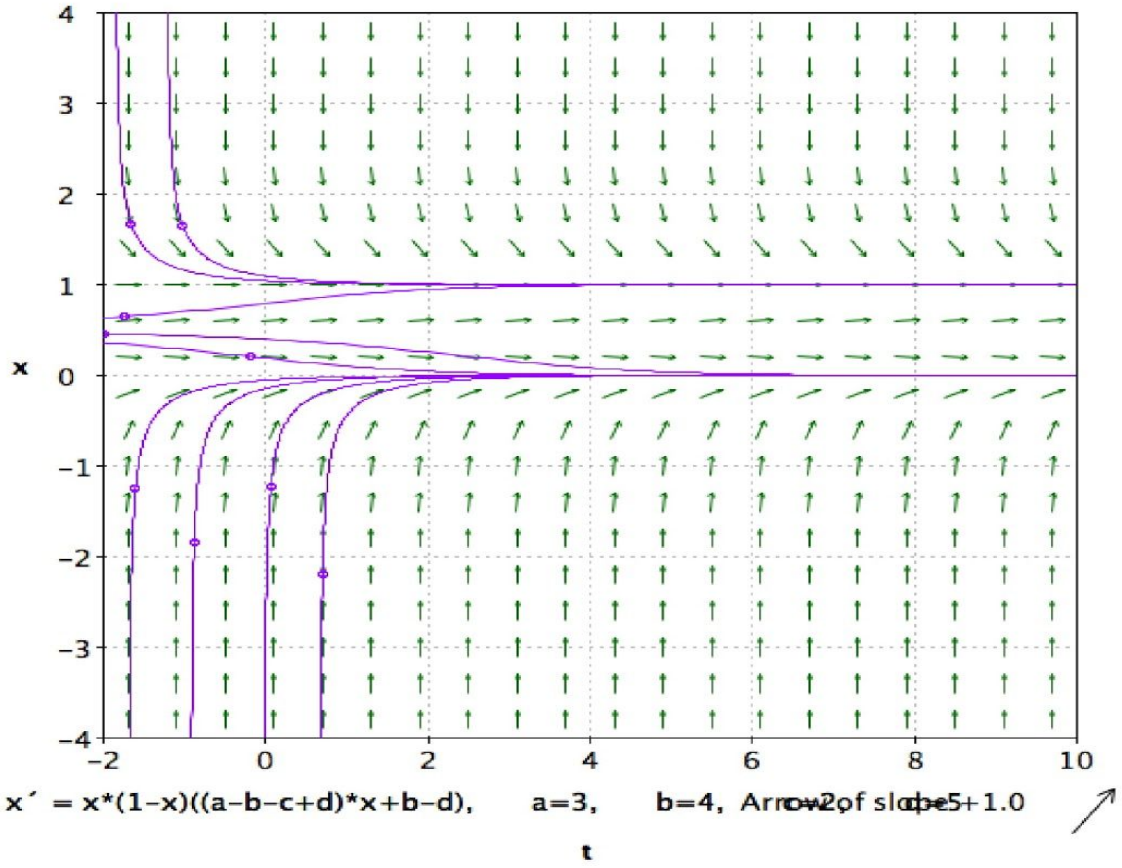
The equation shows the rate of growth, or decay of the respective memes but the actual dynamic occurs on the straight line segment[6].
In this simple situation there are four outcomes. To make things concrete I talk in terms of a leave meme competing with a remain meme.

1. If strategy Leave (say) dominates strategy Remain then Leave is a sink (Evolutionary Stable Equilibrium-ESS) and the flow moves from Remain to Leave.
2. The converse. Remain dominates Leave. Remain is the ESS.
3. If the accelerating, or payoff for both strategis is positive then a so called bistable dynamic with an unstable central equilibrium occurs trapped between the two ESS (L and R). This is the EU Ref situation. The optimal strategy for the two meme is for each to adopt the same strategy as the other. By this is meant that LL interaction and RR interaction deliver the strong payoffs as compared with LR or RL thus the dynamic pushes the two players apart.
4. Stable coexistence where the payoff for both strategies is dominated by the “breaking” effect pulling both strategies back toward an equilibrium. There is a stable central equilibrium and L and R are both unstable. The underlying strategy here is to play the opposite strategy when confronted with the opposing meme. L begets R and R begets L . This is the same dynamic as the median voter theorem(Nash Equilibrium iat the Median Voter point is typically established by the recursive elimination of dominated strategies which drives both players towards the EES) In a best response analysis the best move initially is to do the opposite to one’s opponent

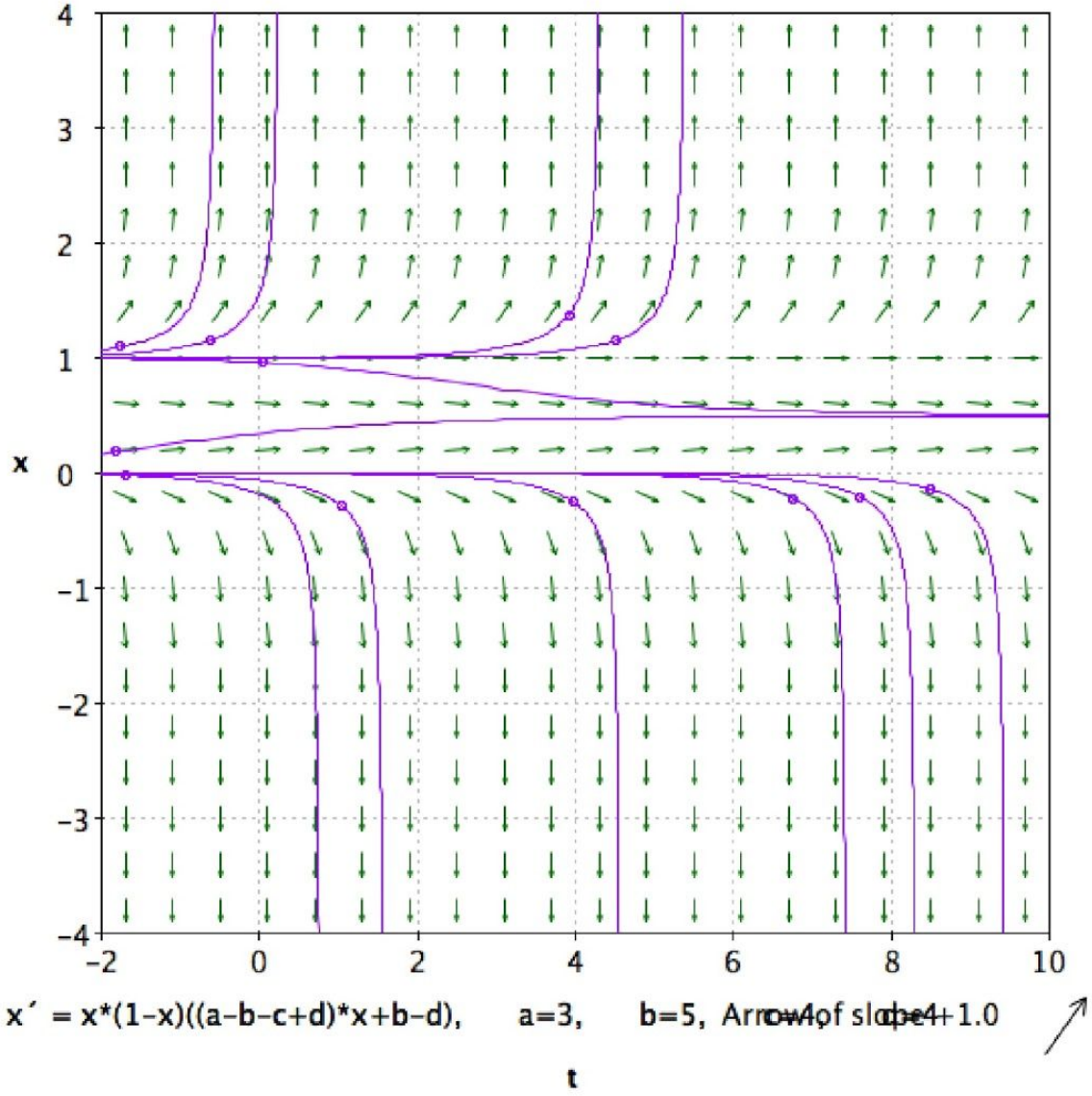
[6] This is a technical requirement

The difference between 3. And 4. Can be illustrated as time plots of the replicator dynamic with differing payoffs.

Situation 3. Bi-stability Compare with the plot of poll results in the first diagram above.



Situation 4. The compromise MVT plot. Note the convergence to an equilibrium.


4
3
2
1
x
0
−1
−2
−3
−4
−2
0
2
4
6
8
10
x´ = x*(1−x)((a−b−c+d)*x+b−d),
a=3,
b=5,
t

Finally here is a picture, (phase portrait) of the dynamic, the important thing is the flow along the line and the equilibrium points. Two stable points at either end of the 01 line with an unstable equilibrium point in the centre. The flows are being sucked towards the extremes. Note this just a theoretical calculation the unstable equilibrium is much more central with real data.